1 Solving the Swing Equation for a network of 3 generators during a fault and post fault

The problem to be considered involves the following: 3 Generators are connected to a small power grid network. The network admittance matrix relating the node voltages (phasors) to the currents (phasors) from the generators is given by Y matrix ($\bar{I} = Y\bar{V}$). Three such matrices are involved in this project: $Y_{prefault}$, Y_{fault} and $Y_{postfault}$. The elements of these matrices are expressed as complex numbers $Y_{ik} = G_{ik} + jB_{ik} = |Y_{ik}| \angle\theta_{ik}$. Each of the generators maintains the magnitude of its excitation voltage E_i for the duration of such faults or small load changes at prefixed value. However the angle of these excitation voltages δ_i (remember this voltage is a phasor and has magnitude and angle) relative to a reference voltage on the grid constantly adjusts to changing load/faults etc.. During transients if these changes in the angles of the generators does not stabilize to a fixed value post-transient, then the grid becomes unstable and the generators loose synchronism with the grid and are usually tripped out by circuit breakers. A cascade of such events can lead to local grid failure. Electrical code fixes the time by which faults must be cleared before this happens and the goal of the project is to study this time. P_e, the electrical power, into the network from each generator is given by

$$P_{ei} = E_i^2 G_{ii} + \Sigma_{k=1,k\neq i}^{k=3} E_i E_k Y_{ik} \cos(\theta_{ik} - \delta_i + \delta_k) \qquad i = 1, 2, 3 \tag{1}$$

The mechanical power P_m into the generators is assumed constant at the turbines input power level as changing a hydraulic turbine power level takes a few minutes whereas electrical network transients and stability issues occur in seconds. The initial angles δ_i of each generator before the fault can be found from the input turbine power level balancing the electrical power out of the generator as given by (1) from $Y_{prefault}$ network matrix and the generator excitation voltages E_i and the knowledge of at least one angle say δ_1 . The simplified equations of motion for each generator is given by:

$$\frac{2H_i}{\omega_i}\frac{d\omega_i}{dt} + D_i\omega_i = P_{mi} - P_{ei} \tag{2}$$

$$\frac{d\delta_i}{dt} = \omega_i - \omega_R \tag{3}$$

where $\omega_R = 377$ rad/s in the North American grid.

2 Problem to considered

1. A fault occurs on the network and the Y_{fault} matrix holds in this time. The minimum time in which the circuit breakers can clear this fault is 5 cycles of the power supply frequency 60 Hz ($= 0.083$ s). After the fault is cleared the $Y_{postfault}$ matrix is the network admittance matrix. Compute the three angles δ_i and plot $\delta_2 - \delta_1, \delta_3 - \delta_1$ versus time upto 3 s and see if the generators do not go unstable (i.e the difference in angles swing in a bounded manner and do not keep on growing).

2. If the result of the above is stable then can the fault clearing time be increased to 8 cycles ?

3. what is the largest number of cycles before the fault must be cleared for the network to be stable post fault ?

4. Try to find the North American code recommended fault clearance time to be set into circuit breakers for such a situation. Does this setting make sense to you based on your work on this project.

3 DATA from Reference for your project

$H_i = 25, 6, 3, D_i = 0.001 \quad i = 1..3$. The units of H_i, D_i are s. $P_{mi} = 0.72, 1.63, 0.85$. The units for power and admittance matrices below are in pu based on a 100 MVA base. The excitation voltages for the generators in pu are: $E_i = 1.0566, 1.0502, 1.0170$ and the initial angle of generator 1 is $\delta_1 = 2.2717°$ respectively. Network admittance matrices:

$$Y_{prefault} = \begin{bmatrix} 0.846 - j2.988 & 0.287 + j1.513 & 0.210 + j1.226 \\ 0.287 + j1.513 & 0.420 - j2.724 & 0.213 + j1.088 \\ 0.210 + j1.226 & 0.213 + j1.088 & 0.277 - j2.368 \end{bmatrix} \quad (4)$$

$$Y_{fault} = \begin{bmatrix} 0.657 - j3.816 & 0.000 + j0.000 & 0.070 + j0.631 \\ 0.000 + j0.000 & 0.000 - j5.486 & 0.000 + j0.000 \\ 0.070 + j0.631 & 0.000 + j0.000 & 0.174 - j2.796 \end{bmatrix} \quad (5)$$

$$Y_{postfault} = \begin{bmatrix} 1.181 - j2.229 & 0.138 + j0.726 & 0.191 + j1.079 \\ 0.138 + j0.726 & 0.389 - j1.953 & 0.199 + j1.229 \\ 0.191 + j1.079 & 0.199 + j1.229 & 0.273 - j2.342 \end{bmatrix} \quad (6)$$

4 Reference

Anderson, P.M. and Fouad, A.A Power System Control and Stability, IEEE-Wiley Press book, 2003, Pgs. 35 - 47. Note: This book can be accessed online through the library using the IEEE Database.